Use of Isolated Problem Approach for the multi-compartment BEM models of Electric Source Imaging

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Abstract—**Isolated Problem Approach (IPA) is a method used in the Boundary Element Method to overcome numerical inaccuracies caused by the high conductivity difference in the skull and the brain tissues in the head. It was previously shown that the source terms can be modified to overcome these inaccuracies for a three layer head model. The integral equations for the general case were also derived when there are an arbitrary number of layers inside the skull. However, the IPA is used in the literature only for three layer head models. Studies that use complex boundary element head models that investigate the inhomogeneities in the brain or model the cerebrospinal fluid do not make use of the IPA. In this study, the generalized formulation of the IPA for multi-layer models is presented in terms of integral equations. The discretized version of these equations are presented in two different forms. Transfer matrix formulation is presented to incorporate the generalized IPA. The accuracy in solutions is tested for a spherical head model. It is observed that, for a radial dipole 1 mm close to the brain surface, the Relative Difference Measure (RDM*) drops from 1.88 to 0.03 when IPA is used.**

Keywords— **boundary element method, electric source imaging, forward problem, isolated problem approach, transfer matrices**

I. INTRODUCTION

Localization of the brain activities using the voltage measurements on the scalp surface (electroencephalography, (EEG)) is called electric source imaging (ESI) [1]-[4]. The *forward problem* of ESI is the solution of the scalp potentials due to a given source configuration. To solve the potential distribution due to electrical sources in the brain various numerical methods can be used. For compartmental models of the head, the Boundary Element Method (BEM) is frequently employed. In the BEM implementations, generally, the three-shell model of the head is used to model the scalp, skull and the brain tissues. However, the low conductivity of the skull layer results in inaccuracy in the solutions of the resultant system of equations. To increase the accuracy, the isolated problem approach (IPA) is employed and a modified set of equations is solved [5], [6]. When the white matter, gray matter, a large ventricle or a tumor in the brain is to be modeled, i.e., when more realistic head models is to be used, the IPA can still be applied. For that purpose, a general formulation was provided [5], however, its numerical implementation has not been attempted by others in this field. In this study, the generalized version of IPA is investigated using spherical head models. The extended forms of the modified source terms in the integral equation are derived. Two different forms of the discretized version are presented to apply the IPA when the BEM is used for a multi-compartment tissue model in the skull.

II. BOUNDARY ELEMENT METHOD

A. Introduction

In a piecewise homogeneous volume conductor model of the head, the electric potential ϕ and the magnetic field due to a current dipole source $\vec{p}$, satisfy the following integral equations [7]:

$$\bar{\sigma}\phi(\vec{r}) = g(\vec{r}) + \frac{1}{4\pi}\sum_{j=1}^{L}(\sigma_j^- - \sigma_j^+)\int_{S_j}\phi(\vec{r}\,')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}\,') \qquad (1)$$

In this equation, S_j , $j = 1,\ldots, L$ represents the boundary surfaces between different conductivity regions. σ_j^- and σ_j^+ represent the inner and outer conductivities of S_j, respectively. $\bar{\sigma}$ is the mean conductivity at the field point, $\vec{R} = \vec{r} - \vec{r}\,'$ is the vector between the field point $\vec{r}$ and the source point $\vec{r}\,'$, and R is the magnitude of $\vec{R}$. The primary source g is defined as shown below:

$$g(\vec{r}) = \frac{1}{4\pi\sigma_0}\frac{\vec{p}\cdot\vec{R}}{R^3}$$

where σ_0 represents the unit conductivity. The second term in (1) can be solved numerically by discretizing the surfaces into elements and computing the surface integrals over these elements [8], [9], [10]. In most studies, the elements are chosen as plane triangles on which the potential has either constant or linear variation. A review of these studies have been reported by Ferguson and Stroink [11]. In this study, triangular, quadratic and isoparametric BEM elements are used for discretizing the surface [12]. Integrating (1) over all elements, a set of equations is obtained. In matrix notation, this can be expressed as:

$$\Phi = \mathbf{g} + \mathbf{C_0}\Phi \qquad (2)$$

where Φ is an Nx1 vector of node potentials and N is the number of nodes in the BEM mesh. $\mathbf{C_0}$ is an NxN matrix whose elements are determined by the geometry and electrical conductivity of the head, and $\mathbf{g}$ is an Nx1 vector representing the contribution of the primary sources. To eliminate the singularity in the solution of (2), the method of matrix deflation is employed [13]. If $\mathbf{I}$ denotes the NxN identity matrix, then

$$\begin{aligned}\Phi &= (\mathbf{I}-\mathbf{C})^{-1}\mathbf{g} \\ &= \mathbf{A}^{-1}\mathbf{g}\end{aligned} \quad (3)$$

Here $\mathbf{C}$ represents the deflated version of $\mathbf{C_0}$.

B. Isolated Problem Approach

In a three-layer head model, when the skull to brain conductivity ratio (β) is small (β<0.1), the solution of (2) yields numerical inaccuracies. The accuracy in solutions can be improved using the Isolated Problem Approach [6]. In the IPA, the region inside the skull is considered as a homogeneous isolated model. The solution to the original set of equations are expressed in terms of the isolated problem solution and a correction term. In this section, 1) the derivation of the integral equations for the correction terms are extended for a multi-layer head model, 2) the discretized forms of the extended modified set of equations are presented.

Let us assume that the head model has L layers. The L^{th} layer is the innermost layer and the $(K-1)^{th}$ compartment corresponds to the low conductivity skull. We will define the surfaces S_1 to S_{K-1} as the *outer* surfaces and S_{K+1} to S_L as *inner* surfaces. The potentials on the inner surfaces are higher than the potentials on the outer surfaces, due to the low conductivity of the $(K-1)^{th}$ compartment. Thus, small numerical errors in the inner surface potentials will be amplified on the outer surfaces. It was shown that these numerical inaccuracies would be reduced if the potential is decomposed as follows [6]:

$$\phi(\vec{r}) = \phi'(\vec{r}) + \phi''(\vec{r}) \quad (4)$$

where $\phi''(\vec{r})$ is the solution of the integral equation for the conductor G bounded by S_K (including surfaces S_K to S_L) and $\phi'(\vec{r})$ is the correction term.

For the isolated problem, $\phi''(\vec{r})$ is zero on the surfaces $S_1,\ldots, S_{K-1}$ and the conductivity $\sigma_K^+ = 0$. Therefore, the corresponding integral equation can be expressed as:

$$\begin{aligned}\bar{\sigma}''(\vec{r})\phi''(\vec{r}) = g(\vec{r}) &+ \frac{\sigma_K^-}{4\pi}\int_{S_K}\phi''(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_K(\vec{r}') \\ &+ \frac{1}{4\pi}\sum_{j=K+1}^{L}(\sigma_j^- - \sigma_j^+)\int_{S_j}\phi''(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}')\end{aligned} \quad (5)$$

where $\bar{\sigma}''$ represents the mean conductivity around a point in the isolated problem space. The integral equation for the correction term $\phi'(\vec{r})$ is obtained by inserting (4) into (1):

$$\begin{aligned}\bar{\sigma}(\vec{r})\phi'(\vec{r}) + \bar{\sigma}(\vec{r})\phi''(\vec{r}) = g(\vec{r}) &+ \frac{1}{4\pi}\sum_{j=1}^{L}(\sigma_j^- - \sigma_j^+)\int_{S_j}\phi'(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}') \\ &+ \frac{1}{4\pi}\sum_{j=K}^{L}(\sigma_j^- - \sigma_j^+)\int_{S_j}\phi''(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}')\end{aligned} \quad (6)$$

Combining equations (5) and (6), the following equation can be obtained for the correction terms on the inner and outer surfaces (i = 1,…, K-1, K+1,…, L):

$$\begin{aligned}(\sigma_i^- + \sigma_i^+)\phi'(\vec{r}) = &\frac{1}{2\pi}\sum_{j=1}^{L}(\sigma_j^- - \sigma_j^+)\int_{S_j}\phi'(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}') \\ &- \frac{\sigma_K^+}{2\pi}\int_{S_K}\phi''(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}').\end{aligned} \quad (7)$$

On the K^{th} surface S_K, $\bar{\sigma}(\vec{r}) = (\sigma_K^- + \sigma_K^+)/2$ and $\bar{\sigma}'' = \sigma_K^-/2$. Thus, for the corrections on the K^{th} surface S_K we obtain

$$\begin{aligned}(\sigma_K^- + \sigma_K^+)\phi'(\vec{r}) = &\frac{1}{2\pi}\sum_{j=1}^{L}(\sigma_j^- - \sigma_j^+)\int_{S_j}\phi'(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}') \\ &- \sigma_K^+\phi''(\vec{r}) - \frac{\sigma_K^+}{2\pi}\int_{S_K}\phi''(\vec{r}')\frac{\vec{R}}{R^3}\cdot d\vec{S}_j(\vec{r}').\end{aligned} \quad (8)$$

Equations (7) and (8) are equivalent to the IPA expressions derived in [5]. By discretizing these integral equations, the following matrix equation can be obtained:

$$\Phi' = \mathbf{g}' + \mathbf{C}\Phi' \quad (9)$$

where Φ' is the Nx1 vector of the correction terms, and $\mathbf{g}'$ is an Nx1 vector representing the modified version of the source term. This equation uses the same $\mathbf{C}$ matrix computed for Φ. Using equations (7) and (8) the modified source term g' can be written in block-matrix form as

$$\mathbf{g}' = \begin{bmatrix}\mathbf{g}'_1 \\ \vdots \\ \mathbf{g}'_{K-1} \\ \mathbf{g}'_K \\ \mathbf{g}'_{K+1} \\ \vdots \\ \mathbf{g}'_L\end{bmatrix} = \begin{bmatrix}\frac{\beta}{\beta-1}\mathbf{C}_{1K}\Phi''_K \\ \vdots \\ \frac{\beta}{\beta-1}\mathbf{C}_{(K-1)K}\Phi''_K \\ \frac{\beta}{\beta-1}\mathbf{C}_{KK}\Phi''_K - \frac{\beta}{\beta+1}\Phi''_K \\ \frac{\beta}{\beta-1}\mathbf{C}_{(K+1)K}\Phi''_K \\ \vdots \\ \frac{\beta}{\beta-1}\mathbf{C}_{LK}\Phi''_K\end{bmatrix} \quad (10)$$

In this representation, $\mathbf{g}'_i$ is a sub-vector of $\mathbf{g}'$ corresponding the ith surface and $\mathbf{C}_{ij}$ is a sub-matrix of $\mathbf{C}$. An alternative representation for $\mathbf{g}'$ can be obtained by multiplying both sides of (5) with $\beta/(\sigma_i^- + \sigma_i^+)$, and discretizing the integral equations as

$$\mathbf{g}' = \begin{bmatrix} \mathbf{g}_1' \\ \vdots \\ \mathbf{g}_{K-1}' \\ \mathbf{g}_K' \\ \mathbf{g}_{K+1}' \\ \vdots \\ \mathbf{g}_L' \end{bmatrix} = \begin{bmatrix} \beta\mathbf{g}_1 + \beta\mathbf{C}_{1(K+1)}\Phi_{K+1}'' + \cdots + \beta\mathbf{C}_{1L}\Phi_L'' \\ \vdots \\ \beta\mathbf{g}_{K-1} + \beta\mathbf{C}_{(K-1)(K+1)}\Phi_{K+1}'' + \cdots + \beta\mathbf{C}_{(K-1)L}\Phi_L'' \\ \beta\mathbf{g}_K - \dfrac{2\beta}{\beta+1}\Phi_K'' + \beta\mathbf{C}_{K(K+1)}\Phi_{K+1}'' + \cdots + \beta\mathbf{C}_{1L}\Phi_L'' \\ \dfrac{\beta}{\beta-1}\mathbf{C}_{(K+1)K}\Phi_K'' \\ \vdots \\ \dfrac{\beta}{\beta-1}\mathbf{C}_{LK}\Phi_K'' \end{bmatrix} \tag{11}$$

where $\mathbf{g}_i$ can be written as

$$\mathbf{g}_i = \frac{2g(\vec{r})}{\sigma_i^- + \sigma_i^+}. \tag{12}$$

For a three layer head model, there is only a single layer inside the skull and K = L = 3. Thus, the expression given in (11) becomes

$$\mathbf{g}' = \begin{bmatrix} \mathbf{g}_1' \\ \mathbf{g}_2' \\ \mathbf{g}_3' \end{bmatrix} = \begin{bmatrix} \beta\mathbf{g}_1 \\ \beta\mathbf{g}_2 \\ \beta\mathbf{g}_K - \dfrac{2\beta}{\beta+1}\Phi_3'' \end{bmatrix} \tag{13}$$

which is the same expression derived previously [6].

C. Accelerated BEM for EEG

In our previous study, we have proposed formulations to calculate the EEG and MEG transfer matrices [14]. In that study, we have derived the accelerated BEM formulations when there is a single inner layer. Using that approach, by pre-calculating and storing relevant matrices, we have achieved a significant decrease in the computation time for a given electrode/sensor configuration. In this study we will generalize the formulation for the EEG transfer matrix assuming arbitrary number of inner layers.

If *m* is the number of electrodes, then the *mx1* vector of electrode potentials can be written as

$$\Phi_e = \mathbf{E}\mathbf{g}' \tag{14}$$

where $\mathbf{E}$ is the *mxN* transfer matrix for the electric field. When the IPA is applied, the right hand side vector $\mathbf{g}$ must be modified using (10). This modification requires Φ_K'' that must be calculated for every source configuration using the isolated model:

$$\begin{bmatrix} \Phi_K'' \\ \vdots \\ \Phi_L'' \end{bmatrix} = \mathbf{A}_s^{-1} \begin{bmatrix} \mathbf{g}_K \\ \vdots \\ \mathbf{g}_L \end{bmatrix} \tag{15}$$

In this equation, $\mathbf{A}_s$ is in the form of $(\mathbf{I}\text{-}\mathbf{C}_s)$ where $\mathbf{C}_s$ is the deflated coefficient matrix for the isolated model. The vector $[\mathbf{g}_K \ldots \mathbf{g}_L]$ is the corresponding source vector. Note that, to compute the node potentials on the outer surface of the isolated model only the first N_K rows of $\mathbf{A}_s^{-1}$ is required (where N_K denotes the number of nodes on that surface).

III. Results

In this section, the accuracy in the numerical solutions obtained using the generalized form of the IPA are tested for a four layer spherical model. The model represents brain, CSF, skull and scalp with conductivities 0.33, 1.0, 0.0042, and 0.33 S/m, respectively [15]. The radii of the spheres are chosen as 61, 65, 71, and 75 mm as described in [5]. The analytical and numerical solutions are compared using the RDM and RDM* [5].

The BEM mesh used in the simulations has 512 elements and 1026 nodes per layer. To improve the accuracy in solutions, the recursive integration technique is employed [16]. Accuracy in the numerical solutions is tested with the analytical solutions provided by [17]. Table 1. shows the percentage RDM and RDM* values for various tangential (x-directed) dipole locations on the z-axis (z=1-6 cm). The numerical solutions are obtained twice (with and without the IPA).

It is observed that application of the IPA improves RDM significantly. For deep dipoles RDM* is relatively small and it is not effected by the application of the IPA. For shallow dipoles RDM* increases if the IPA is not applied. Table 2. presents the same information for radial dipoles. For the radial shallow dipoles, the increase in the RDM and RDM* is evident (Note that the maximum value for RDM* is 2.).

Table 1. The relative difference measures (%RDMs and RDM*s) for various tangential (x-directed) dipoles located on the z-axis (z =1-6 cm) in a 4-layer spherical head model. The results are presented for solutions with and without the IPA.

Distance (cm)	With IPA % RDM	With IPA RDM*	Without IPA % RDM	Without IPA RDM*
1.0	0.50	0.0006	12.0	0.0060
1.5	0.49	0.0009	12.1	0.0062
2.0	0.49	0.0011	12.2	0.0064
2.5	0.49	0.0015	12.2	0.0066
3.0	0.49	0.0018	12.3	0.0068
3.5	0.48	0.0022	12.4	0.0072
4.0	0.48	0.0026	12.6	0.0087
4.5	0.48	0.0031	12.9	0.0141
5.0	0.49	0.0039	13.8	0.0263
5.5	0.53	0.0046	13.9	0.0384
6.0	1.29	0.0118	16.9	0.1170

Table 2. The relative difference measures (% RDMs and RDM*s) for various radial (z-directed) dipoles located on the z axis (z = 1-6 cm) in a 4-layer spherical head model. The results are presented for solutions with and without IPA.

Distance (cm)	With IPA % RDM	With IPA RDM*	Without IPA % RDM	Without IPA RDM*
1.0	0.50	0.0009	11.9	0.0059
1.5	0.54	0.0023	11.8	0.0063
2.0	0.69	0.0050	11.7	0.0081
2.5	1.04	0.0093	11.5	0.0122
3.0	1.61	0.0154	11.3	0.0189
3.5	2.36	0.0232	11.2	0.0282
4.0	3.29	0.0325	11.2	0.0399
4.5	4.34	0.0431	11.7	0.0543
5.0	5.49	0.0545	11.9	0.0770
5.5	6.70	0.0640	26.7	0.2370
6.0	9.98	0.0321	227.9	1.8870

IV. CONCLUSION and DISCUSSION

To increase the accuracy in the forward problem solutions of ESI with the boundary element head models the IPA must be applied. In general, a single layer is assumed in the skull. For realistic head models with arbitrary number of layers in the skull a generalized version of IPA is required. In this study a generalized formulation for the IPA was described. The related integral equations and the discretized version of the modified source terms were presented. In our previous study [14], we have proposed the accelerated BEM approach which improves the solution speed of the EEG and MEG forward problem solutions. The accelerated BEM formulation for the EEG transfer matrix was updated in this paper to account for the generalized IPA formulations. The accuracy obtained by the new formulation was tested with spherical 4-layer models.

Two formulations were derived for the modified source terms in generalized version of the IPA. The matrix equation given in (10) is suitable for a general multi-layer implementation where there are arbitrary number of layers in the skull. The integral equations for this case have also been described in [5]. However, the alternative form (11) is more efficient when there is a single layer inside the skull layer since no sub-matrices ($\mathbf{C_{ij}}$) are needed to be stored and used. The matrix equations of the alternative formulation reduces to the equations given in [6] for a three-layer head model.

For a 4-layer spherical model, it was observed that the application of the IPA improves the RDM for both deep and shallow dipoles. The improvement in RDM*, however, was apparent for only shallow dipoles. For a radial dipole that is 1 mm close to the brain surface, the RDM* dropped from 1.88 to 0.03. The corresponding % RDM decreased from 227.9% to 9.9%.

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