Accelerated BEM for the Calculation of the potential and magnetic fields

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ABSTRACT

Localization of neural electrical activity in the brain using magnetoencephalography (MEG) and electroencephalography (EEG) measurements is called electro-magnetic source imaging (EMSI). For accurate solution of EMSIs a numerical approach such as the Boundary Element Method (BEM) must be employed. In this study, an advanced realistic head model is used for solving the forward problem (FP) of EMSI using the BEM. The main drawback of using realistic head models is the computation time to solve the FP. Typically inverse problem algorithms require a large number of FP solutions. Thus, the speed of FP solutions directly affect the localization speed. To overcome this problem, two formulations are proposed to compute the transfer matrices for EEG and MEG that map the sources to the sensor fields. Once these matrices are calculated, very fast FP solutions can be obtained using only matrix multiplications. For a quadratic BEM mesh with 9680 nodes individual FP solutions take 138 ms for 256 EEG electrodes and 197 ms for 256 MEG sensors on a Pentium IV 2.4 GHz computer.

KEY WORDS

Accelerated BEM, transfer matrix, realistic head model, EEG, MEG.

INTRODUCTION

Electrical activity inside the human brain can be localized from potential (EEG) and magnetic field (MEG) measurements. Source localization consists of two sub-problems. Calculating the potential distribution on the scalp and magnetic field distribution near the scalp surface due to a given source configuration is the forward problem. Finding the neural sources from the measured fields is the inverse problem. To increase the accuracy in the source localization, realistic head models are constructed using numerical methods. In this study, the Boundary Element Method (BEM) is employed for solving the forward problem [Gençer, 1999]. An advanced BEM implementation that uses quadratic, isoparametric elements is used. The implementation can handle intersecting tissue boundaries and the realistic meshes have about 10000 unknowns [Akalın, 2002]. When such complex models are used for source localization, the solution speed of the forward problem becomes a critical factor. Since the inverse problem algorithms require many forward solutions, even a small increase in the forward problem computation time becomes significant. A BEM formulation yields a matrix equation which must be solved to obtain the unknown potentials. For solving the matrix equation, first the BEM coefficient matrix, is factorized or inverted reducing the solution time for subsequent forward solutions. One obvious way to further reduce the precomputation and solution times is to solve the fields only at the sensor positions. This can be done by computing a transfer matrix between the source distribution and the sensor fields. Several researchers have exploited this idea for EEG [Cuffin, 1995], [Fletcher, 1995], [Fuchs, 2001]. However, a method for computing a transfer matrix for the solution of the magnetic fields at the sensor locations has not yet been proposed. This study proposes a new approach for the calculation of both potential and magnetic fields using only matrix-vector multiplications after calculating the necessary transfer matrices.

BEM SUMMARY

The BEM is a widely used numerical method to calculate the electric potential due to an electrical activity in the head [Barr, 1966]. In this study, the related surface integrals are calculated numerically by dividing the surface into isoparametric quadratic elements [Gençer, 1999]. If the potential is to be calculated at M nodes, then in matrix notation, it is possible to obtain the following equation:

$$\mathbf{\Phi} = \mathbf{A}^{-1}\mathbf{g} \tag{1}$$

where $\mathbf{\Phi}$ is an $M\times 1$ vector of node potentials, $\mathbf{A}$ is an $M\times M$ matrix whose elements are determined by the geometry and electrical conductivity of the layers, and $\boldsymbol{g}$ is an $M\times 1$ vector representing the contribution of the primary sources. To eliminate the singularity in $\mathbf{A}$, the method of matrix deflation is employed [Lynn, 1968]. Isolated Problem Approach (IPA) is implemented to overcome numerical errors caused by high conductivity difference around the skull layer [Hämäläinen, 1989]. Once $\mathbf{\Phi}$ is computed, $\mathbf{B}$ is calculated using the potential values. This can be written in matrix notation as follows:

$$\mathbf{B} = \mathbf{B}_o + \mathbf{H\Phi} \tag{2}$$

If there are n magnetic sensors, $\mathbf{B}$ is an $n\times 1$ vector representing the the magnetic field at the sensor locations. $\mathbf{B}_0$ is the $n\times 1$ vector of magnetic fields at the same sensor locations for an unbounded homogenous medium (primary magnetic field). Here, $\mathbf{H}$ is an $n\times M$ coefficient matrix determined by the geometry and electrical conductivity of the head. The second term in equation (2) is called the secondary magnetic field.

ACCELERATED BEM FOR EEG

Solution of equation (1) provides the potential at all nodes. $\mathbf{\Phi}$ can be solved with iterative algorithms such as conjugate gradient, which are relatively slow, or by directly inverting or factorizing the $\mathbf{A}$ matrix, which is computationally very expensive. In the inverse problem solution, however, only the potential field at the electrode positions $\mathbf{\Phi}_e$ is required. For m electrodes, $\mathbf{\Phi}_e$ is an $m\times 1$ vector and can be expressed as:

$$\mathbf{\Phi}_e = \mathbf{D}\mathbf{A}^{-1}\mathbf{g} = \mathbf{E}\mathbf{g} \tag{3}$$

Here, $\mathbf{D}$ is a matrix composed of 0's and 1's and chooses the relevant rows of $\mathbf{A}^{-1}$. Thus if the selected rows of $\mathbf{A}^{-1}$ are calculated and stored, then $\mathbf{\Phi}_e$ can be calculated by a simple matrix-vector multiplication. Let $\mathbf{E}$ be the transfer matrix for potential field calculations, it is defined as $\mathbf{E}=\mathbf{D}\mathbf{A}^{-1}$. Then, since $(\mathbf{A}^T)^{-1} = (\mathbf{A}^{-1})^T$, we can write $\mathbf{A}^T\mathbf{E}^T=\mathbf{D}^T$. Therefore we can construct the $\mathbf{E}$ matrix row-by-row using $\mathbf{A}^T\mathbf{e}_i=\mathbf{d}_i$ where $\mathbf{e}_i$ and $\mathbf{d}_i$ are the *ith* rows of $\mathbf{E}$

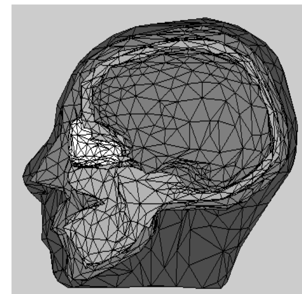
and **D** respectively using *m* matrix solutions. Once **E** is precomputed, forward problem solutions are obtained using matrix-vector multiplications. When IPA is used, the **g** vector must be modified. **g** is modified using the potential field, $\mathbf{\Phi}_S$, on the homogeneous surface of the modified layer [Hämäläinen, 1989]. For this purpose, the inverse of a sub-matrix $\mathbf{A}_S$ which is the coefficient matrix of the the model having only the modified layer must be computed and stored. Then, $\mathbf{\Phi}_S$ can be computed with a vector-matrix muliplication of $\mathbf{A}_S$ and $\mathbf{g}_S$.

ACCELERATED BEM FOR MEG

Note that caculation of the magnetic field using equation (2) requires the knowledge of the potential at all nodes. It is possible to express the magnetic field without explicity computing $\mathbf{\Phi}$. The second term on the righthand-side of equation (2) can be written as:

$$\mathbf{B}_S = \mathbf{H\Phi} = \mathbf{HA}^{-1}\mathbf{g} = \mathbf{Mg} \qquad (3)$$

Where $\mathbf{B}_S$ is the $n{\times}1$ vector representing the secondary magnetic field. **M** is the $n{\times}M$ transfer matrix for the magnetic field calculations. The transfer matrix and can be computed row-by-row by applying the same technique used in the calculation of **E**. When IPA is used, the **g** vector must be modified as discussed above. This modification requires the inverse of the $\mathbf{A}_S$ sub-matrix to be computed and stored.



RESULTS

The computational complexity is measured for a realistic head model. The realistic model that is used in this study is shown in Figure 1. The mesh consists of 9680 nodes, 4864 second-order elements and contains the following tissue types: Scalp (0.2 S/m), skull (0.005 S/m), CSF (1 S/m), brain (0.2 S/m) and eyes (0.5 S/m). The computational cost of various stages of forward problem solutions are given in Table 1. The solutions are obtained using a 2.4 GHz Pentium IV computer with 1 GB memory. The pre-computation time to calculate the potential field and the magnetic field takes 3.4 hours. After these pre-computations, a single potential field solution takes only 138 msec. and magnetic field solution takes 197 msec using IPA.

Table 1. Computational complexity for a realistic mesh with 9680 nodes, the modified layer matrix has 2618 nodes.

Matrix filling (**A** matrix, 9680 x 9680)	12 min.
Single matrix solution ($\mathbf{A}^T\mathbf{e}_i=\mathbf{d}_i$)	3 min.
Calculation of the transfer matrix for EEG (**E**) for 256 electrodes	3.2 hours
Matrix filling ($\mathbf{A}_s$ matrix, 2618 x 2618)	2 min.
Calculation of the inverse of $\mathbf{A}_s$ matrix	23 sec.
Matrix filling (**H** matrix) for 256 sensors	28 sec.
Calculation of the transfer matrix for MEG (**M**) for 256 sensors	3.2 hours
Calculation of the modified right hand side (RHS)	10 ms
Calculation of the potential field ($\mathbf{\Phi}_e$) for 256 electrodes	138 ms
Calculation of the magnetic field (**B**) for 256 sensors	197 ms

DISCUSSION

In this work, two new formulations are proposed to calculate the potential and magnetic fields efficiently. After pre-calculating the relevant matrices, it is possible to obtain vary fast forward problem solutions. The accelerated BEM approach is employed to obtain fast forward problem solutions required by the inverse problem algorithms. This work extends the accelerated BEM approach to calculate the magnetic field and it is also improved to use the IPA for both EEG and MEG solutions. Using this BEM implementation it is possible to create complex head models and obtain very fast forward solutions. Thus, it is possible to use global iterative inverse problem algorithms such as the genetic algorithms which require a large number of forward solutions.

REFERENCES

Akalın Z, Gençer NG. Investigating the Effects of Eye Conductivity on EMSI Forward Problem Using a Realistic BEM Head Model. Proc. of International Journal of Bioelectromagnetism 2002;4:219-220.

Barr RC, Pilkington TC, Boineau JP, Spach MS. Determining surface potentials from current dipoles, with application to electrocardiography. IEEE Trans. Biomed. Eng. 1966;13:88–92.

Cuffin NB. A method for localizing EEG sources in realistic head models. IEEE Trans. Biomed. Eng. 1995;42(1):68-71.

Fletcher DJ, Amir A, Jewett DL, Fein G. Improved method for computation of potentials in a realistic head shape model. IEEE Trans. Biomed. Eng. 1995;42(11):1094-1104.

Fuchs M, Wagner M and Kastner J. Boundary Element Method Volume Conductor Models for EEG Source Reconstruction. Clin. Neur. 2001;112:1400-1407.

Gençer NG, Tanzer IO. EEG Forward problem solution of electromagnetic source imaging using a new BEM formulation with high-order elements. Phys. Med. Biol. 1999;44:2275-2287.

Hämäläinen MS, Sarvas J. Realistic Conductivity Geometry Model of the Human Head for Interpretation of Neuromagnetic Data. IEEE Trans. Biomed. Eng. 1989;36(2):165-171.

Lynn MS, Timlake WP. The use of multiple deflations in the numerical solution of singular systems of equations, with applications to potential theory. SIAM J. Numer. Anal. 1968;5:303-322.